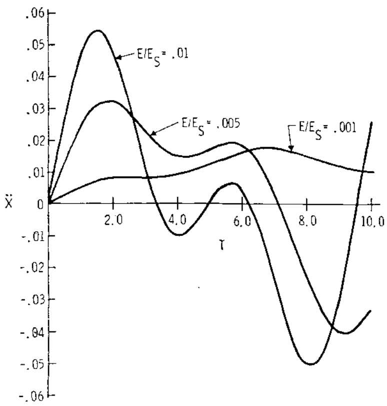

Fig. 2 Rigid cylinder acceleration.

$$\delta_1 = [4(1-\nu)/(3-4\nu)\Gamma][2-(5-4\nu)k^{-2}+(3-4\nu)k^2] \quad (6g)$$

$$\delta_2 = [4(1-\nu)/(3-4\nu)\Gamma] \times [2+(1-4\nu)k^{-2}-(3-4\nu)k^2] \quad (6h)$$

$$\sigma_3 = \sigma_1 - \sigma_2,\ \tau_3 = \tau_1 - \tau_2,\ \delta_3 = \delta_1 - \delta_2 \quad (6i)$$

$$k = a/b,\ \Gamma = (1-k^{-2})\{(3-4\nu)(1+k^2)\ln k + [1/(3-4\nu)](1-k^2)\} \quad (6j)$$

Equations (5) are solved for x by taking a Laplace transform over t with zero initial conditions. The transform variable and the transform of x are denoted by s and $\bar{x}$, respectively. Then the transformed solution for the rigid cylinder response is

$$\bar{x} = -bb_0(s^2+b_1)\bar{p}(s)/s^2(s^4+b_2s^2-b_3) \quad (7a)$$

where

$$b_0 = \frac{G\delta_1}{2a^2\gamma\rho hb},\ b_1 = \frac{K(\delta_1-\delta_2) - bG(\delta_2\tau_1-\delta_1\tau_2)}{\rho hb^2\delta_1} \quad (7b)$$

$$b_2 = [2K + bG(\sigma_1+\tau_2) - (\rho hb^2G\delta_3/a^2\gamma)]/\rho hb^2 \quad (7c)$$

$$b_3 = (G/a^2\gamma\rho^2h^2b^3)\{2\rho hbK\delta_3 + \rho hb^2G[\delta_3(\sigma_1+\tau_2) - \delta_1\sigma_3 - \delta_2\tau_3] + a^2\gamma[K(\tau_3-\sigma_3) - bG(\sigma_1\tau_2-\sigma_2\tau_1)]\} \quad (7d)$$

The time dependence of the load is taken as

$$p(t) = PH(t) \quad (8)$$

where P is the magnitude of the pressure and $H(t)$ is the Heaviside unit function. A Laplace transform of Eq. (8) is taken, and the result is substituted into Eq. (7a). Then Eq. (7a) is inverted by standard techniques and the rigid cylinder response x is twice differentiated with respect to time to give the acceleration

$$\ddot{x} = \frac{-bb_0P}{(m^2-n^2)}\left[\frac{(m^2+b_1)}{m^2}\cosh mt - \frac{(n^2+b_1)}{n^2}\cosh nt\right] - \frac{bb_0b_1P}{m^2n^2} \quad (9)$$

where $\pm m$, $\pm n$ are the roots of $s^4 + b_2s^2 - b_3 = 0$.

Results

The applied pressure time dependence is assumed to be a rectangular pulse of duration t_d. For this loading, the rigid cylinder acceleration $\ddot{x}$ is easily obtained from Eq. (9) by the principal of superposition. The dimensionless variables $\ddot{X} = \ddot{x}/\ddot{x}_{RB}$ and $T = ct/b$ are introduced, where

$$\ddot{x}_{RB} = bP/2(a^2\gamma + 2\rho hb),\ c = [E_s/\rho(1-\nu_s^2)]^{1/2} \quad (10)$$

$\ddot{x}_{RB}$ is the acceleration of the system of Fig. 1 if it were considered to be a rigid body. Curves of $\ddot{X}$ vs T are presented in Fig. 2 for $\nu = 0.10$, $\nu_s = 0.25$, $\gamma/\rho = 0.50$, $a/b = 0.25$, $b/h = 40$, $T_d = 0.07$, and $E/E_s = 0.01, 0.005, 0.001$. Figure 2 quantitatively demonstrates the reduction of the rigid cylinder acceleration $\ddot{X}$ with decreasing core Young's modulus E.

References

[1] Payton, R. G., "Dynamic Membrane Stresses in a Circular Elastic Shell," *Journal of Applied Mechanics*, Vol. 28, Sept. 1961, pp. 417–420.

[2] Humphreys, J. S. and Winter, R. W., "Dynamic Response of a Cylinder to a Side Pressure Pulse," *AIAA Journal*, Vol. 3, No. 1, Jan. 1965, pp. 27–32.

[3] Alzheimer, W. E., Forrestal, M. J., and Murfin, W. B., "Transient Response of Cylindrical, Shell-Core Systems," *AIAA Journal*, Vol. 6, No. 10, Oct. 1968, pp. 1861–1866.

[4] Goodier, J. N. and McIvor, I. K., "The Elastic Cylindrical Shell Under Nearly Uniform Radial Impulse," *Journal of Applied Mechanics*, Vol. 31, June 1964, pp. 259–266.

Errors in Freestream Reynolds Number of Helium Wind Tunnels

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Nomenclature

A^*/A = ratio of effective throat-to-nozzle cross-sectional area
C_{Re_∞} = correction factor for freestream Reynolds number
M = Mach number
p = pressure, atm
q = dynamic pressure, atm
Re = unit Reynolds number, ft^{-1}
T = temperature, °K
μ = coefficient of viscosity, g/cm-sec

Subscripts

$t,1$ = reservoir stagnation conditions
$t,2$ = stagnation conditions behind normal shock
∞ = freestream conditions

Introduction

FOR nearly two decades, hypersonic fluid dynamic studies have been performed in wind tunnels employing helium as the flow medium. One of the primary reasons for using helium is that it liquifies at a very low temperature; hence, high Mach numbers can be generated in a wind tunnel without preheating the helium prior to expansion. Conventional-type wind tunnels employing helium at ambient supply temperature have operated at Mach numbers of approximately 10–26. For this Mach-number range, the corresponding freestream static temperature range is about 9°–1.5°K. Recently, Mach numbers of 30–70 were generated in the Langley hotshot tunnel, using helium as the test gas, in which calculated freestream static temperatures ranged from approximately 0.8°–12°K (Ref. 1). Heretofore, it has been common practice to use the empirical expression for viscosity of Keesom[2] in determining the freestream Reynolds number at these low temperature conditions (for example, see Refs. 3–5). However, a recent survey by one of the authors[6] revealed that usage of Keesom's expression for viscosity at these temperatures results in freestream Reynolds number errors up to 65%. The purpose of this Note is

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to: 1) bring this matter to the attention of those associated with helium wind tunnels as well as those using helium data involving freestream Reynolds number, 2) present more accurate expressions for helium viscosity at temperatures down to 0.2°K, 3) provide correction factors that can be applied readily to existing freestream Reynolds numbers calculated using Keesom's expression for viscosity, and, for convenience, 4) present simple expressions for determining freestream flow parameters, which account for real-helium (compressibility) effects, in terms of commonly measured flow quantities.

Helium Viscosity at Low Temperatures

At the time Keesom formulated an empirical expression for viscosity, only a single experimental data point was available at temperatures below 4°K, and this point (at $T = 1.64$°K) deviated from the extrapolation of the curve fit applied to the other higher temperature data. In the mid 1950's, more comprehensive experimental helium viscosity data were published for temperatures in the range of 1.3° to 4°K [7,8] these data are shown in Fig. 1, where viscosity is plotted as a function of temperature. Also shown in Fig. 1 are the empirically based predictions of Keesom[2] and Keyes[9]; both predictions are observed to overestimate the experimental viscosity data. In 1956, Keller[10] presented calculations of helium (He^4) viscosity, based on quantum-mechanical theory of transport phenomena, for temperatures from 0 to 40°K. Keller found that the second-order approximation for viscosity, as calculated by using quantal collision integrals based on the Lennard Jones (12–6) intermolecular potential, was in good agreement with the available experimental data at these low temperatures. However, Keller presents his low-temperature results with some reservations, stating they are qualitative only. A somewhat more extensive theoretical study of helium transport properties, at the low temperatures of interest, was performed more recently by Monchick et al.[11]. The quantal collision integrals of Ref. 11 also were based on the Lennard Jones (12–6) potential, and the calculated viscosity was observed to be in close agreement with that of Keller. Values of the second-order approximation for viscosity, as determined from the quantal collision integrals tabulated in Ref. 11, are shown in Fig. 1. The greater accuracy provided by these calculations based on quantum-mechanical theory, in comparison to the empirically based predictions, is clearly evident at temperatures below 4°K.

Because of the complexity associated with the calculation of collision integrals, curve fits have been applied to viscosity calculations (based on the collision integrals of Ref. 11) to obtain relatively simple expressions for viscosity. For temperatures above 8°K, it is recommended that Keesom's empirical expression, given by

$$\mu = (5.023 \times 10^{-6})T^{0.647} \quad (1a)$$

be used. (As stated in Ref. 2, this expression is believed to be accurate to within 1% for temperatures up to 1100°K.) The expression obtained from curve fitting the viscosity calculations for temperatures 3.6° to 8°K is

$$\mu = (-1.5691 + 3.4167T - 1.0317 \times 10^{-1}T^2) \times 10^{-6} \quad (1b)$$

for temperatures 1.2° to 3.6°K is

$$\mu = (5.0200 - 3.2241T + 2.0308T^2 - 2.2351 \times 10^{-1}T^3) \times 10^{-6} \quad (1c)$$

and for temperatures 0.2° to 1.2°K is

$$\mu = (2.1630 - 2.6665 \times 10^1T + 1.2054 \times 10^2T^2 - 1.8741 \times 10^2T^3 + 1.2682 \times 10^2T^4 - 3.1823 \times 10^1T^5) \times 10^{-6} \quad (1d)$$

Expressions (1b–d) provide values of viscosity to within 1% of those calculated using the collision integrals of Ref. 11.

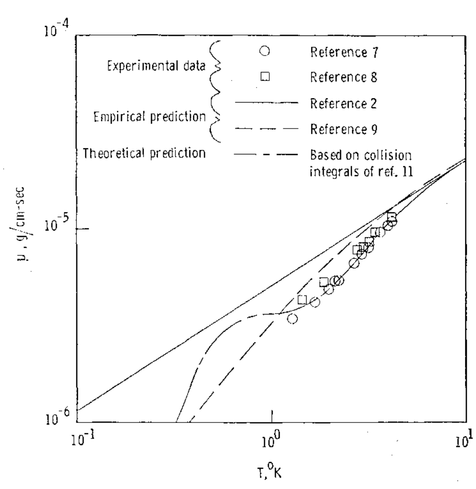
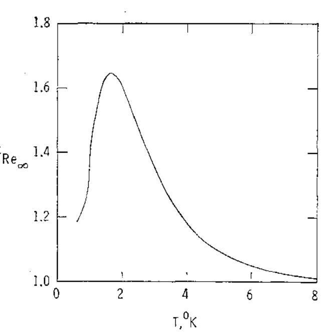


Fig. 1 Viscosity of helium at low temperatures.



T,°K	C_{Re_∞}
0.6	1.187
0.8	1.227
1.0	1.384
1.2	1.528
1.4	1.616
1.6	1.645
1.8	1.636
2.0	1.602
2.4	1.500
2.8	1.395
3.2	1.306
4.0	1.181
5.0	1.095
6.0	1.050
7.0	1.026
8.0	1.011

Fig. 2 Correction factor for freestream Reynolds number of helium as a function of temperature.

Correction Factor for Reynolds Number

In many instances, the freestream Reynolds number in a helium wind tunnel has been determined from the ideal-helium charts of Refs. 3 and 4. The values of Reynolds number in Refs. 3 and 4 are based on Keesom's expression for viscosity and thus are in appreciable error for freestream static temperatures below about 5°K. To provide a rapid means of correcting the Reynolds number obtained from Refs. 3 and 4, correction factors C_{Re_∞}, {ratio of Reynolds number based on viscosity, calculated by using Eqs. (1b–d), to that based on Keesom's expression for viscosity [Eq. (1a)]} were determined and are shown plotted (and tabulated) in Fig. 2 for a temperature range of 0.6°–8°K. The more accurate Reynolds number is obtained by simply multiplying the value of Reynolds number obtained from Ref. 3 or 4 by the correction factor C_{Re_∞}.

Real-Helium Flow Expressions

Since several helium wind tunnels operate at reservoir conditions where real–helium (compressibility) effects are significant, it is often desirable to have explicit expressions for freestream flow parameters that account for these effects. Such relations have been derived using the simplified method for determining real–helium hypersonic flow parameters presented in the appendix of Ref. 5. (In Ref. 5, the virial form of the equation of state was used to determine real–helium flow parameters for reservoir pressures to 3600 atm and temperatures from 300° to 15,000°K.) These relations, expressed in terms of measured quantities $p_{t,1}$, $T_{t,1}$, and $p_{t,2}$ are given by

$$M_\infty = 2.6825C_1/(p_{t,2}/p_{t,1})^{0.342} \text{ for } p_{t,2}/p_{t,1} \geq 2.5 \times 10^{-3} \quad (2a)$$

$$M_\infty = 2.796C_1/(p_{t,2}/p_{t,1})^{0.335} \text{ for } p_{t,2}/p_{t,1} < 2.5 \times 10^{-3} \quad (2b)$$

$$p_\infty = (15.5885p_{t,1}C_2)/(M_\infty^2 + 3)^{2.5} \quad (3)$$

$$T_\infty = (3T_{t,1}C_3)/M_\infty^2 + 3 \quad (4)$$

$$q_\infty = 0.833p_\infty M_\infty^2 \quad (5)$$

$$A^*/A = (16M_\infty C_4)/(M_\infty^2 + 3)^2 \quad (6)$$

where, from Ref. 5, the correction factors are

$$C_1 = 1 + p_{t,1}[(0.5114/T_{t,1}^{1.3744}) - (1.1937 \times 10^2/T_{t,1}^{2.5901})] = 1 + 1.5833 \times 10^{-4}p_{t,1}$$

$$C_2 = 1 + p_{t,1}[(1.4538/T_{t,1}^{1.3640}) - (1.3801 \times 10^3/T_{t,1}^{2.8233})] = 1 + 4.7498 \times 10^{-4}p_{t,1}$$

$$C_3 = 1 + p_{t,1}[(0.7901/T_{t,1}^{1.3628}) - (2.4311 \times 10^1/T_{t,1}^{2.1286})] = 1 + 2.0581 \times 10^{-4}p_{t,1}$$

$$C_4 = 1 + p_{t,1}[(1.7301/T_{t,1}^{1.3776}) - (5.3966 \times 10^2/T_{t,1}^{2.6027})] = 1 + 4.8359 \times 10^{-4}p_{t,1}$$

[The second form of these correction factors applies for ambient reservoir temperature ($T_{t,1} = 295°K$)]. The correction factors C_2 and C_3 are accurate to within 0.2% for Mach numbers above 20, but C_2 may be as much as 1% too large and C_3 0.5% too large for Mach numbers between 10 and 20. The correction factor C_4, although not presented in Ref. 5, was obtained at the same time as the other factors and has an uncertainty of about 1.5% for all conditions. (The correction factor C_1 is independent of Mach number.) For ideal-helium, $C_1 = C_2 = C_3 = C_4 = 1$. The freestream Reynolds number, per foot, is given by

$$Re_\infty = 8.7479 \times 10^3(M_\infty p_\infty/T_\infty^{1/2}\mu_\infty) \quad (7)$$

where μ_∞ is found from Eq. (1). The system of units for the equations in this note is given in the nomenclature.

References

[1] Miller, C. G., III, "Hotshot Tunnel Operations With Helium at Mach Numbers in Excess of 30," Proposed NASA TN.

[2] Keesom, W. H., *Helium*, Elsevier, Amsterdam, 1942, pp. 104–107.

[3] Mueller, J. N., "Equations, Tables, and Figures for Use in the Analysis of Helium Flow at Supersonic and Hypersonic Speeds," TN 4063, 1957, NACA.

[4] Mueller, J. N., "Ideal-Gas Tables for Helium Flow in the Mach Number Range from 40 to 100," TN D-1252, 1962, NASA.

[5] Miller, C. G., III and Wilder, S. E., "Real-Helium Hypersonic Flow Parameters for Pressures to 3600 Atmospheres and Temperatures to 15,000°K," TN D-4869, 1968, NASA.

[6] Maddalon, D. V. and Jackson, W. E., "A Survey of the Transport Properties of Helium at High Mach Number Wind Tunnel Conditions," Proposed NASA TM.

[7] Becker, E. W., Misenta, R., and Schmeissner, F., "Die Zähigkeit von gas förmigem He^3 and He^4 zwischen 1.3°K und 4.2°K," *Zeitschrift für Physik*, Vol. 137, 1954, pp. 126–136.

[8] Coremans, J. M. J. et al., "The Viscosity of Gaseous He, Ne, H_2, and O_2 Below 80°K," *Physica*, Vol. 24, 1958, pp. 557–576.

[9] Keyes, F. G., "A Summary of Viscosity and Heat Conductivity for He, A, H_2, O_2, N_2, CO, CO_2, H_2O, and Air," *Transactions of the ASME*, Vol. 73, No. 5, July 1951, pp. 589–596.

[10] Keller, W. E., "Calculation of the Viscosity of Gaseous He^3 and He^4 at Low Temperatures," *The Physical Review*, Vol. 105, 1956, pp. 41–45.

[11] Monchick, L. et al., "Transport Properties of Gaseous He^3 and He^4," *The Physical Review*, Vol. 139, No. 4A, Aug. 1965, pp. A1076–A1082.

Response of a Circular Elastic Shell to Moving and Simultaneous Loads

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Introduction

Much of the existing experimental information on the response of impulsively loaded, re-entry vehicle-type structures has been obtained by testing with sheet explosives; e.g., see Ref. 1. For many applications the goal of an impulse simulation is to impart a simultaneously applied, short duration, pressure pulse to the structure. However, sheet explosives impart an impulse by means of a short duration, pressure pulse which travels at the detonation velocity of the explosive. Thus, rather than providing the desired simultaneously applied impulsive load, the structure is loaded by a traveling, short duration, pressure pulse. The accuracy of simulating simultaneously applied impulsive loads with sheet explosives has recently been investigated by Florence[2] for beams and by the authors[3] for ring structures. In these papers, a comparison is made between the response produced by traveling forces which represent detonation waves in the sheet explosive and the response produced by simultaneously applied impulsive loads. Results presented in Refs. 2 and 3 can be used to evaluate the use of sheet explosives to simulate simultaneously applied impulsive loads.

In Ref. 3, a formula for the membrane stress in a long, circular, elastic shell produced by two identical forces, traveling from $\theta = 0$ to $\theta = \pm\pi/2$, is derived. Numerical results are presented for the membrane stresses produced by simultaneous, impulsive loads applied over one-half the shell circumference and two concentrated forces that move from $\theta = 0$ to $\theta = \pm\pi/2$ at a constant velocity V. It is shown in Fig. 4 of Ref. 3 that both loadings produce nearly the same response for a cosine distributed impulse and forces whose magnitude vary as the cosine function. The explosive loading technique modeled in Ref. 3 consists of wrapping variable thickness, sheet explosive one-half way around the structure and detonating the explosive with an exploding bridgewire which extends down the length of the cylinder.

In this Note, the accuracy of a similar impulse simulation technique for structures with circular cross sections is evaluated. Briefly, this method consists of wrapping sheet explosive one-half way around the structure and detonating the explosive at $\theta = -\pi/2$. Then the structure is loaded by a short duration, pressure pulse which travels from $\theta = -\pi/2$ to $\theta = +\pi/2$ at a constant velocity V. A detailed explanation of this method is presented by Lindberg in Ref. 4.

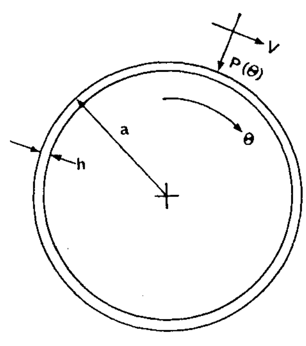

Fig. 1 Geometry of the problem.

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